Supervisor Expression of Interest
MSCA - Marie Sklodowska Curie Action - (PF) Postdoctoral Fellowship 2022

Supervisor name:	**Michele Correggi**
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Department Name:	Department of Mathematics
Research topic:	Mathematical Physics
MSCA-PF Research Area Panels:	☐ CHE_Chemistry ☐ ECO_Economic Sciences ☐ ENG_Information Science and Engineering ☐ ENV_Environmental and Geosciences ☐ LIF_Life Sciences ☒ **MAT_Mathematics** ☐ PHY_Physics ☐ SOC_Social Sciences and Humanities
Politecnico di Milano Areas:	☐ Cultural Heritage ☐ Smart Cities X **Horizon Europe Missions** ☐ Health ☐ Industry 4.0
Brief description of the Department and Research Group (including URL if applicable):	The candidate will be involved in the growing group of mathematical physicists at the Department of Mathematics, and specifically on the research line about the mathematical features of quantum mechanics. The group consists of the supervisor, who's also the group leader, a researcher – Marco Falconi (https://www.mfmat.org/) – and 3 Ph.D. students – Andrea Calignano, Michele Fantechi and Filippo Perani. Further hirings are planned in the near future, since the group is of extremely recent formation (the leader was hired only in 2020) but plays an important and strategic role in the research plan of the Department.

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Title	**Strongly Correlated Quantum Systems**

Brief project description:
(max 1 page)

The rigorous derivation of effective theories for large systems is a very influential and topical subject for the mathematical physics community in the last 20 years. In the typical picture, a large quantum system is investigated in a suitable regime, and it is proven that an effective description in terms of a few- or even one-body (nonlinear) model is approximately correct. So far, almost only systems which are asymptotically uncorrelated were taken into account, namely the correlations between different particles are either absent or vanish in the limit. This is the case of the mean-field regime of interacting bosons but also of the Gross-Pitaevskii limit, where correlations have a non-trivial effect but disappear in the large number of particle limit.
However, in order to describe more realistic and physically relevant system, the existence of correlations, forcing to various degrees a departure from the independent particles picture has to be taken into account. We plan to address this point by studying the role of correlations in different quantum systems and developing mathematical techniques to rigorously investigate the properties of such systems.

1. The hard-core Bose gas is a model for a Bose-Einstein condensate for which an infinitely strong repulsive interaction at short range forces a strongly correlated phase to emerge at low temperature. An amount of properties of the hard-core Bose gas are conjectured but not yet rigorously proven, among them the derivation of the Lee-Huang-Yang formula as an upper bound to the ground state energy. We aim at addressing this problem and specifically investigating the properties of the Jastrow function, as a paradigmatic trial state for the model. Related open problems are the study of the excitation spectrum in the thermodynamic limit and, ultimately, the proof of Bose-Einstein condensation.
2. Another quite intriguing, correlated quantum system is an interacting Bose gas trapped by multiple wells. It was indeed recently shown [3] that, if the tunneling between wells is negligible, then a strongly correlated phase emerges in the ground state, causing a suppression to the fluctuation of some N-body observables. While a central limit theorem argument, typically applicable to trapped Bose gases, would predict fluctuations at scale of order of the square root of the particle number, a regime of suppressed tunneling fluctuations occurs at a strictly smaller scale. We will investigate various unexplored aspects of this phenomenon, among which the sharp behavior of reduced fluctuations and the possible emergence of a central limit theorem at a new scale, the persistence of reduced fluctuations along the time evolution, and the extension to more singular regimes than the currently known mean-field.
3. Finally, correlations are extremely important in investigating the interaction of quantum particles with quantized fields. Typical physical models describing such a phenomenon are the Nelson model and the polaron model [2], in which a particle in interaction with a quantized bosonic field becomes self-interacting at low energies and large coupling. We plan to focus first on the unconfined polaron, which is a translation-invariant model whose classical limit (the Pekar functional) exhibits a self-trapping mechanism that breaks translation invariance. The interplay between those features is not completely understood yet, as quantum minimizing sequence might or might not lose mass. Our goal is to exploit the recently devoted semiclassical techniques for systems with infinitely many degrees of freedom to shed light on this question.

References
[1] M. Correggi, M. Falconi. *Ann. Henri Poincaré* **19**, 189-235 (2018).
[2] A. Olgiati, N. Rougerie, D. Spehner. Preprint *arXiv:2101.08690* (2021).